

THE TRACKING STUDY OF A LOW ENERGY PROTON SYNCHROTRON*

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Abstract Linear and nonlinear space charge effects are studied using the lattice of the SSC low energy booster synchrotron as an example. A new feature of the multiparticle tracking used in this work is the self-consistent charge distribution in both transverse and longitudinal directions. Time-consuming calculations with many particles ($\approx 5,000$) to simulate the charge distribution are made possible by the use of supercomputers. The emittance growth of the beam as a whole instead of an amplitude growth of a few selected particles is emphasized in this study.

INTRODUCTION

The emittance growth in the low energy synchrotron is believed to be caused mainly by space charge effects. To estimate space charge effects, Laslett tune shift formula is usually invoked.¹ The tune of each particle goes down because of the space charge defocusing and may cross various resonances. The tune shift, however, depends on the amplitude of each particle in the beam, producing the tune spread. In the Laslett formula, the longitudinal line density is represented simply by a bunching factor, but this is a function of time due to the space charge force itself.

Some beam manipulations such as H^- injection, adiabatic capture or the so-called painting scheme make the beam behavior immediately after the injection quite complicated and the analytical approach for this problem is not very promising. In this paper we will describe the development of a multiparticle tracking program and present some results. The program can handle space charge effects in a self-consistent manner independent of any particular charge distribution. The tracking would be very time-consuming for a realistic lattice and for adequate turn numbers but it is still feasible with supercomputers such as CRAY X-MP or NEC SX-2 (which was used for this study).

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TRACKING PROCEDURES

The tracking is based on the computer code TEAPOT.² In TEAPOT, each element is treated as a combination of thin lens and drift space. We introduce the space charge force into the program as a thin lens kick. The tracking routine consists of two parts, space charge kick at certain time intervals and multiparticle tracking between space charge kicks. The strength of the space charge force is recalculated at each kick from the updated beam distribution. One feature of this tracking is that the independent variable is *time* instead of distance along the design orbit. We should note that the space charge force acts on the beam itself continuously and it should not have the periodicity of the machine lattice. Another advantage of taking *time* as an independent variable is the ease of incorporating the longitudinal beam dynamics with or without acceleration. As the time interval for space charge kicks, we take 10^{-8} seconds which corresponds to 140 space charge kicks around the ring.

EVALUATION OF SPACE CHARGE

We express the beam potential as $P(x, y, s) = V(x, y) * R(s)$, where $V(x, y)$ should have a zero potential at the beam pipe radius. $R(s)$ is equivalent to the longitudinal line density. The separation of the potential should be a good approximation for a proton beam as the bunch length is long compared with the transverse beam size. Once the beam potential is found, the transfer matrix that represents the space charge kick can be obtained in a straight forward manner. However, because of the limitation of computer time, we introduce further simplifications. The first is to simplify the equation by taking $R(s)$ to be a step function. Then the partial derivative with respect to s becomes zero everywhere except at the boundary. Physically this is equivalent to ignoring the longitudinal space charge but the dependence of the transverse space charge force on the longitudinal position is retained. The partial derivatives with respect to x and y can be easily obtained when the transverse distribution is expanded in terms of Hermite polynomials.³ We take the first three of the Hermite polynomials to reconstruct an arbitrary transverse beam distribution. In order to include the longitudinal space charge force as a function of transverse coordinate, axial symmetry for the transverse distribution is used with Gaussian $V(r)$. For the longitudinal space charge, the arbitrary longitudinal distribution $R(s)$ is expanded in Fourier series which is terminated near the cut off frequency of the beam pipe.⁴ The dependence of the transverse position was included by the integration of the transverse force using

Exponential-Integral function.

TRACKING RESULTS

As the test lattice, we took the SSC low energy booster synchrotron.⁵ The machine is designed for 600 MeV injection with 0.75π mm – mr normalized rms transverse emittance and 72 bunches, each bunch containing 10^{10} particles. The lattice consists of normal FODO cells and has the superperiodicity two. The operating point is (11.84, 11.78). For the nonlinear fields in dipoles, we assumed random sextupole errors that drive resonances $3\nu_x = 35$ and $\nu_x + 2\nu_y = 35$. These two resonance lines have the stopband width of 1.1×10^{-3} . There are chromaticity correcting sextupoles but they do not affect the resonances because of their periodic arrangement.

Figure 1 shows the number of particles in a bunch and the vertical emittance after 128 turns with and without the longitudinal space charge. Figure 2 is its evolution as a function of turn number when the longitudinal space charge is included. Each beam has the same initial emittance on both transverse and longitudinal planes ($\epsilon_{x,y} = 0.75\pi$ mm – mr, $\epsilon_s = 0.37\pi$ mm – rad, normalized rms) the transverse beam shape is matched to the acceptance which is calculated without space charge effects. The initial distribution is Gaussian but particles beyond 3σ were not used in the tracking. The RF voltage applied from the beginning is such that the stationary bucket height is five times the beam momentum spread. The synchronous phase is always zero.

Beyond a certain number of particles in a bunch, the emittance increases more or less linearly with the number of particles. This agrees qualitatively with the data taken at Fermilab.⁶ To explain these results, it is generally believed that there is a density limit in the transverse phase space which is related to the allowed tune shift. In terms of the Laslett tune shift, the results here show that the maximum tune shift is about -0.35 to -0.45. The relation of the final emittance and the number of particles in a bunch looks almost the same in both cases. The longitudinal emittance increases by less than 10% when the longitudinal space charge is included.

However, for a beam with ten times smaller longitudinal emittance, the result is different. The higher line density in a bunch creates the large longitudinal space charge and the longitudinal emittance increased as well. The rapid increase in both transverse and longitudinal emittance is shown in Figs. 3 and 4. With this small initial longitudinal emittance, the final longitudinal emittance increases more or less linearly with the number of particles as shown in Fig. 5. Figure 6 is the result

of two identical cases, one with and the other without longitudinal space charge effects. It clearly demonstrates the importance of including the longitudinal effect in a self-consistent manner.

Figure 7 shows the final emittance and the number of particles when the initial transverse emittance is one-third of the previous case. When the intensity is low, the initial emittance is maintained. However, as the beam intensity is increased, the emittance goes up in the same way as in the previous case. One interesting point is that the final emittance as a function of the number of particles was slightly less. Because of the larger longitudinal space charge, even if the initial longitudinal emittance is the same, the slight longitudinal emittance growth occurs and this suppresses the transverse growth.

CONCLUDING REMARKS

We have investigated space charge effects in a self-consistent manner with the tracking program which allows arbitrary charge distributions. The result shows the reasonable relation of the emittance and the number of particles in a bunch under various conditions. Some cases indicate a continuing emittance growth after 128 turns. More tracking studies are necessary for this cases in order to make reliable conclusion.

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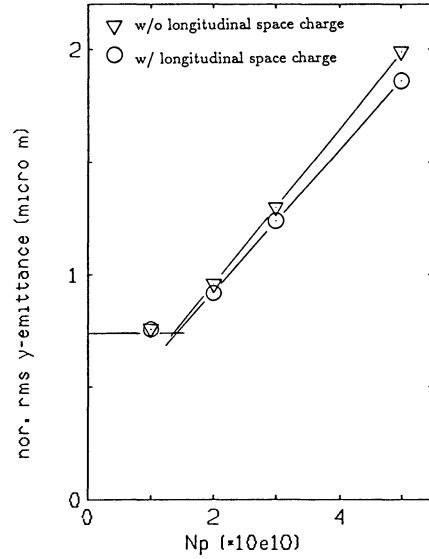


Fig 1: Number of particles in a bunch vs. final transverse emittance.

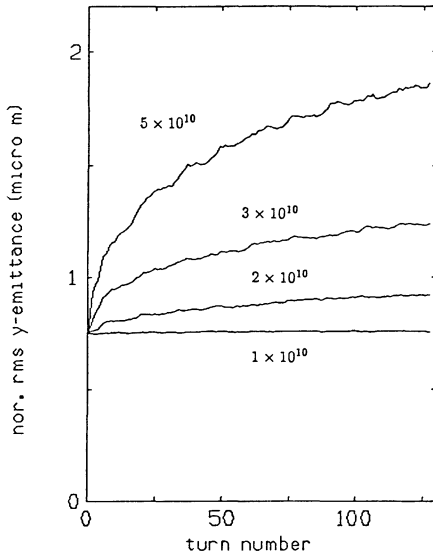


Fig 2: Evolution of transverse emittance with longitudinal space charge.

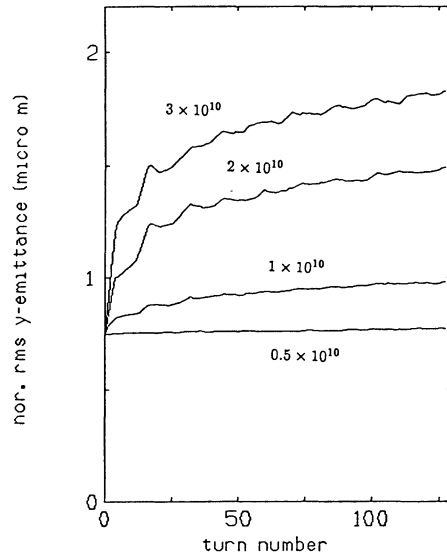


Fig 3: Evolution of transverse emittance when the initial longitudinal emittance is small.

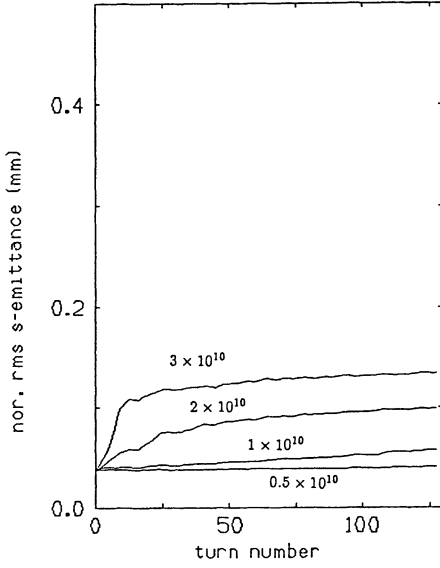


Fig 4: Evolution of longitudinal emittance when the initial longitudinal emittance is small.

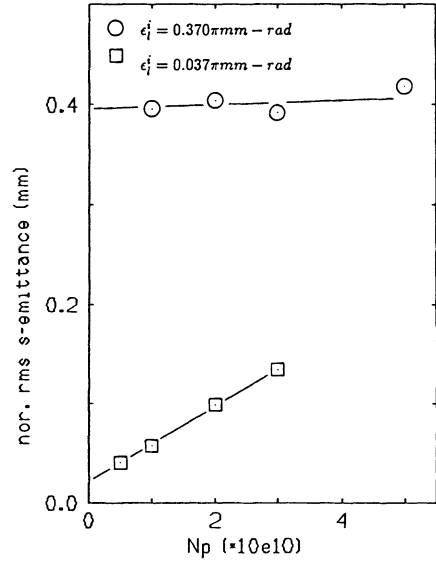


Fig 5: Number of particles in a bunch vs. final longitudinal emittance.

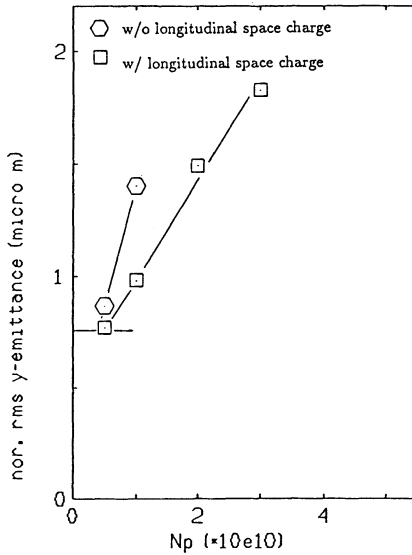


Fig 6: Number of particles in a bunch vs. final transverse emittance when the initial longitudinal emittance is small.

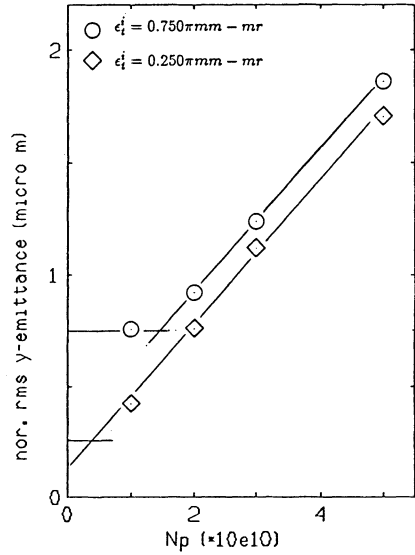


Fig 7: Number of particles in a bunch vs. final transverse emittance for two initial transverse emittances.